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USE OF WEIGHT FUNCTIONS TO DETERMINE THE STRESS INTENSITY FOR A--ETC(U)
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REPORT

MRL-R-814

USE OF WEIGHT FUNCTIONS TO DETERMINE THE STRESS INTENSITY FOR A CRACKED THICK-WALLED CYLINDER

Graham Clark

Approved for Public Release

MAY, 1981

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16. ABSTRACT (if this is security classified, the announcement of this report will be similarly classified):

The use of weight functions to determine stress intensities for cracked specimens is relatively simple, and, while the initial derivation of a suitable weight function for some specimen geometries may be complex, the technique offers substantial benefits. In particular, stress intensities may be estimated for any initial stress distribution, and variations in stress intensity which result from the introduction of residual stress fields may be handled without the need to perform a full re-analysis for each case. The stress intensity at the tip of a crack which grows through a known stress field has been estimated using a weight function technique. The geometry considered is a thick-walled cylinder (Ro/Ri = 1.8) containing a longitudinal crack along the bore surface, and stress distributions corresponding to a variety of loading arrangements are used as examples.

C O N T E N T S

USE OF WEIGHT FUNCTIONS TO DETERMINE THE STRESS INTENSITY FOR A CRACKED THICK-WALLED CYLINDER

1. INTRODUCTION

Fracture mechanics methods have advanced in recent years to the stage where the behaviour of a crack which has grown into stressed material may be predicted with reasonable accuracy, given a knowledge of the crack tip environment and, in particular, the stresses which are developed in the material surrounding the crack tip. These stresses are usually the major factor influencing fracture behaviour at the crack tip and for linear elastic conditions are conveniently represented by the stress-intensity factor (K). The concentrated stress normal to the crack plane σ_{yy} at distance r from the tip is given by

$$\sigma_{yy} = K/\sqrt{2\pi r} \qquad (1)$$

When K reaches a critical value K_c (the material's fracture toughness) unstable crack extension occurs; this value may be determined directly by a standard procedure, or may be estimated from other fracture parameters using a variety of techniques, and plays a major role in fracture safety considerations.

One of the major practical problems associated with fracture mechanics analyses is that of relating the macroscopic conditions of applied load, specimen geometry and crack length to the stress intensity which controls crack tip behaviour. This is particularly important in view of the need to relate critical crack tip conditions ($K = K_c$) to the combination of applied load and crack length which produces these conditions, and which must be evaluated in order to minimise the risk of unstable crack growth in practical engineering situations.

Various techniques are available for determining this relationship (the stress intensity calibration) for each particular geometry; for example, analytical methods which are usually limited to relatively simple configurations, experimental techniques which may be expensive in terms of

labour and material, and numerical techniques such as finite element methods which are extremely versatile but require considerable computing time. In general, when a number of initial stress distributions in an uncracked body have to be considered, the whole stress intensity calibration procedure must be repeated, and analysis of situations in which, say, any one of a number of residual stress distributions may be present can involve considerable effort.

One technique, however, simplifies the calculation of stress intensities in such situations by introducing an intermediate step, namely the calculation of a function by which point loads along the crack may be weighted before their contributions to the stress intensity at the crack tip are summed. Because any distribution of point loads may be used, the desired stress distribution is easily represented, and knowledge of a weight function makes it possible to evaluate the stress intensity by means of only one integration.

The weight function method is described below, and the function is evaluated for the important case of a radial crack in a thick-walled pressure vessel. Stress intensity calibrations for various loading arrangements are calculated to check the weight function and to illustrate its use.

2. WEIGHT FUNCTIONS

Weight function methods have been described in detail by various authors [1-4]; however the principle may be described briefly using a Green's function approach. Fig. 1(a) represents a body of arbitrary shape containing a crack; the body is subjected to external forces which produce a stress intensity K_A, and the crack surface is necessarily free of normal stresses. This is equivalent to the sum of two cases; an identical, stressed body with no crack and hence no stress intensity (but with stresses $\sigma(x)$ at the latent crack position Fig. 1(b)) and an identical cracked body, (Fig. 1(c)) in which there is no external force, but an internal pressure P(x), equal to $\sigma(x)$, in the crack. The stress intensity K_B produced by the internal pressure must equal K_A; and therefore if the stress distribution in an uncracked, loaded, body is known, we may determine the stress intensity for a crack which is introduced in that body by considering the crack to contain an internal pressure equal to that stress distribution [1,5].

This result is important in that the superposition principle may be used; an internal pressure $\sigma(x)$, as shown in Fig. 2 may be considered to be the sum of a series of point loads F at position t such that

$$F(t) = \sigma(t) \,.\, dt \qquad (2)$$

each of which contributes dK towards the whole stress intensity,

$$dK = F(t) \,.\, G(t) = \sigma(t) \,.\, G(t) \,.\, dt \qquad (3)$$

where G(t) is a weight function representing the way in which the response of

the crack system (i.e. stress intensity) varies with different inputs (forces at different positions along the crack).

Hence, the total stress intensity at the crack tip is

$$K = \int_{-a}^{+a} \sigma(t) \cdot G(t) \cdot dt \qquad (4)$$

which may be integrated numerically (or otherwise) for any $\sigma(t)$.

In practice, the weight function is known for various simple geometries; for more complex systems, it may be determined [1] from estimates of crack surface displacements, or more directly using finite element techniques.

3. CRACKED RING GEOMETRY

The geometry of interest here is a section of an infinitely long thick-walled pressure vessel with a wall ratio (Ri/Ro) of 0.555, as shown in Fig. 3. This contains a longitudinal crack growing radially from the bore and represents a straight-fronted fatigue crack growing in the wall of a 76 mm gun barrel. This barrel, in practice, contains residual compressive stresses in the bore region which are produced by a swage autofrettage process. Due to the need to consider the effects of various residual stress distributions, the weight function technique is the most appropriate for estimating the stress intensities associated with fatigue cracks in the barrel.

Stress-intensity calibrations for cracked rings with Ro/Ri of 1.25 to 2.50 (in 0.25 increments), under internal and crack surface pressure have been determined using a modified mapping collocation technique by Bowie and Freese [6]. A calibration for the geometry used here, with Ro/Ri = 1.8018, estimated from these results by linear interpolation was used to check the accuracy of the weight function.

The weight function is used here in a form compatible with expressions for the stress intensity produced by a point load F acting on the surface of a crack in an infinite body. Using the notation of Fig. 3, this is [7]

$$dK = \frac{2a}{\sqrt{\pi a}} \; \frac{F}{\sqrt{a^2 - t^2}} \qquad (5a)$$

which, using equation (2) and introducing the weight function for the finite geometry, becomes

$$dK = \frac{2a}{\sqrt{\pi a}} \; \frac{\sigma(t)}{\sqrt{a^2 - t^2}} \cdot G\left(\frac{t}{a}\right) \cdot dt \qquad (5b)$$

and, integrating

$$K = \frac{2}{\sqrt{\pi}} \int_0^1 \sigma(t/a) \cdot G(t/a) \cdot \frac{a}{\sqrt{1 - t^2/a^2}} \, d(t/a) \qquad (6)$$

A two-dimensional finite element representation of the specimen, with crack lengths $0.02<a/T<0.9$, where T is the ring wall thickness, was used to determine the weight function $G(t/a)$. The mesh and program have been described in detail elsewhere [8], and the procedure used was to apply an arbitrary load to a suitable node on the crack surface, and to calculate the resultant dK in equation (5), thus deriving a value for the weight function G. This is equivalent to assuming that $\sigma(t/a)$ has the form of a delta-function i.e.

$$\sigma(t/a) = P\delta\left((t-t')/a\right) \qquad (7)$$

so that the integral of equation (6) becomes

$$K = \frac{2}{\sqrt{\pi}} \int_0^1 P.\delta\left((t-t')/a\right) \cdot G(t/a) \cdot \frac{a}{\sqrt{1 - t^2/a^2}} \, d(t/a) \qquad (8)$$

$$= \frac{2P}{\sqrt{\pi}} G(t'/a) \frac{a}{\sqrt{1 - (t'/a)^2}} \qquad (9)$$

from which $G(t'/a)$ is evaluated from the finite-element analysis value of K. Using this approach, the weight function was evaluated for the majority of crack surface node positions and for a range of crack lengths.

The weight function for $a/T = 0$ is that for an edge crack in an infinite half-plane, and has been published elsewhere [9] in the form of a tenth-order polynomial in (t/a). It was more convenient to use a fourth-order fit to data derived from the original polynomial:

$$G(\tfrac{a}{T} = 0) = 1.29475 + 0.0006075 \left(\tfrac{t}{a}\right) - 0.799316 \left(\tfrac{t}{a}\right)^2 + 0.748105 \left(\tfrac{t}{a}\right)^3 - 0.244171 \left(\tfrac{t}{a}\right)^4 \qquad (10)$$

Similarly, for point loads applied at the crack tip, the weight function is unity; these limiting values were all used with those from the finite element analysis as data for a program which fitted a surface to $(t/a, a/T)$ in order that the weight function could be evaluated for cracks of any length within the range considered. This surface represents the way in which the weight function varies along the crack as the crack extends towards the back face of the specimen.

The surface fit used is a bicubic spline [10-13]

$$G(t/a,a/T) = \sum_{IJ} C(I,J)*MI(t/a)*NJ(a/T) \quad (11)$$

where coefficients C(I,J), (I = 1,2,3,4, J = 1,2,3,4,5) are listed in Table 1 and MI, NJ are normalised cubic B-splines with one internal knot at a/T = 0.32. The surface, shown in Fig. 4, is not a particularly good approximation to the data obtained from the finite element calculations, probably because the distribution of data points in the (t/a,a/T) plane was far from uniform, but the errors involved (up to ±3%) are acceptable for most applications. For more critical work, polynomials were fitted to the G values for each crack length for which a stress intensity determination was required; however, in contrast to use of the weight function surface, stress intensities for only a limited number of crack lengths could be determined with this method.

4. USE OF THE WEIGHT FUNCTION

The stress intensity calibrations for various applied loading configurations were determined by integrating equation (6) using a suitable function σ for the stress across the crack plane in the uncracked body. Two examples are given here, the first of which served to check the results of the weight function determination.

4.1 Internal Pressure on Bore and Crack Surfaces

Bowie and Freese [6] have published stress-intensities for thick-walled cylinders which contain a single radial crack growing from the bore. The loading used represented internal pressure applied to the cylinder bore and to the crack surface, and represents an extremely important practical fracture mechanics problem. By linear interpolation of these results, a calibration for Ri/Ro = 0.555 was generated; this is estimated to be accurate to within approximately 2% (the original data was claimed to be accurate to 1%). To use the weight function technique, the tangential stress σ_h for a pressurized cylinder at radius R with full crack face pressure P is

$$\sigma_h = P \cdot \left[(Ro/R)^2 + 1\right] \Big/ \left[(Ro/Ri)^2 - 1\right] + P \quad (12)$$

and was used in equation (6), with integration performed using increments in t/a of 0.00003.

The results obtained using the weight function are compared with those of Bowie and Freese [6] in Fig. 5; agreement is worst near a/T = 0.1 and 0.9, approaching 5% at these points, as would be expected from the difficulties involved in generating a finite element mesh when the crack tip approaches a free surface. However, the agreement between the curves is generally good, indicating that the weight function technique is capable of producing stress intensity calibrations of an accuracy suitable for most applications.

The major advantage of using weight functions becomes apparent when the case of a partially-pressurized crack is considered; this can occur in practice when the crack is sealed by a corrosion product or is partially closed by residual stresses. All that is necessary is to replace the second term on the right hand side of equation (12) with the appropriate crack face pressure distribution and perform the integration in equation (6) as before.

4.2 Cracked Ring in Compression

The difficulties involved in growing fatigue cracks in the laboratory by hydraulic pressurization have resulted in the increasing use of a ring compression specimen described by Jones [14] and shown in Fig. 6; this configuration involves compressing a ring between two platens, and monitoring the growth of a fatigue crack from a notch on the bore surface towards the loading point, and has been used successfully to determine fatigue crack growth rates [15,16] and stress intensity calibrations derived from these growth rates [16]. A stress intensity calibration for a ring with $Ri/Ro = 0.555$ is available from both experimental [15] and finite element [8] sources, and this calibration is compared in Fig. 7 with that derived from the weight function technique. The stress distribution for an uncracked ring in compression is required and in this case was determined from the work of Ripperger and Davids [17], who published solutions for the stresses both on and normal to the loading line in thick walled cylinders.

The weight function was used in the form of the surface shown in Fig. 4, and the calibration shown in Fig. 7 contains small errors which are nevertheless clearly acceptable for most purposes; the agreement between the curves is good, and could be improved by the use of a polynomial representation of the weight function, as described above.

5. CONCLUSION

The use of a weight function technique for the determination of stress-intensity calibrations has been demonstrated, and the weight function has been determined for a thick-walled cylinder containing a radial crack. The weight function may be used with any crack plane stress distribution for the uncracked ring (such as a residual stress distribution) to determine the stress intensity at the tip of a crack introduced in the cylinder.

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TABLE 1

COEFFICIENTS C(I,J) IN EQUATION (11)

J \ I	1	2	3	4
1	1.267905	1.209342	1.127912	1.003005
2	0.9893884	1.035266	0.814600	0.9918211
3	2.111450	1.798387	1.867205	1.042516
4	2.744155	2.496006	0.8610213	0.9420078
5	3.858993	3.555081	3.641089	1.016016

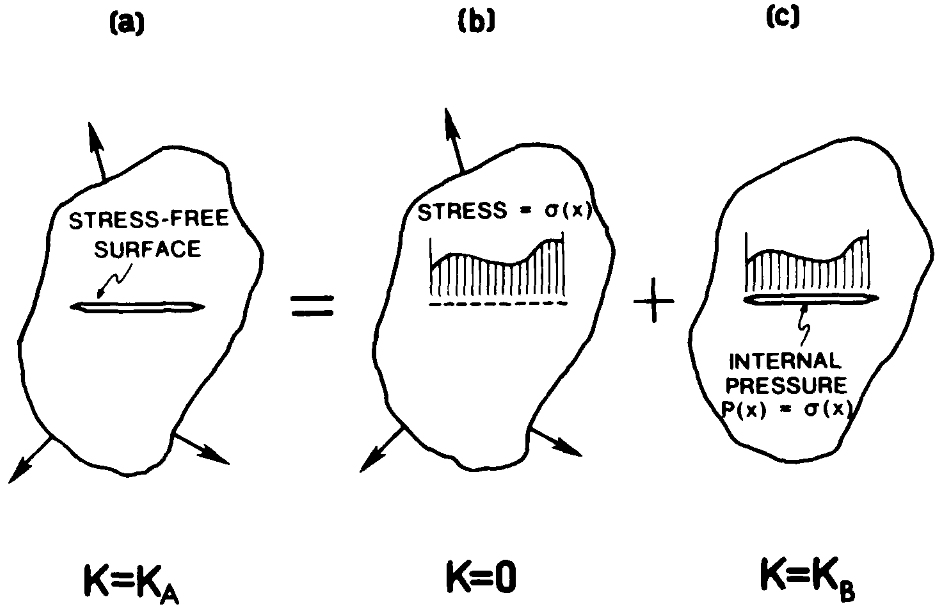

FIG. 1 - The stress intensity for a cracked, externally loaded body equals that for the same body in which the crack is subjected to an internal pressure distribution. This pressure distribution is identical to the crack plane stresses produced in an uncracked body by the external loading.

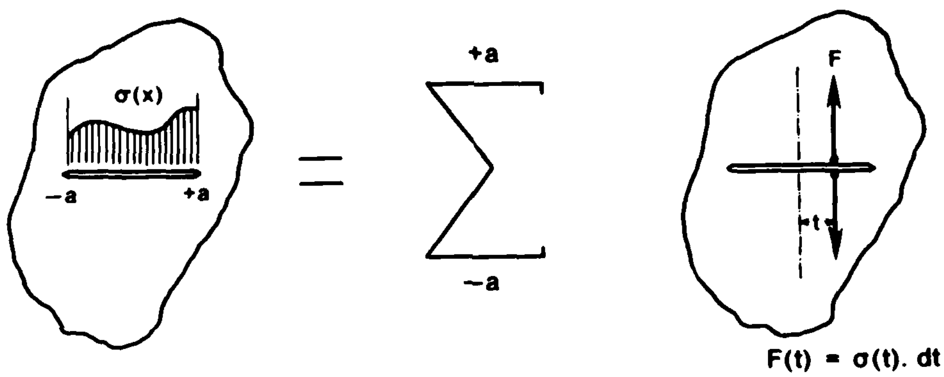

FIG. 2 - Representation of a continuous pressure distributon $\sigma(x)$ as the sum of a series of point loads. $F(t) = \sigma(t)\ dt$, and the stress intensity for the pressurized crack equals the sum of the stress intensities for the point-loaded configurations.

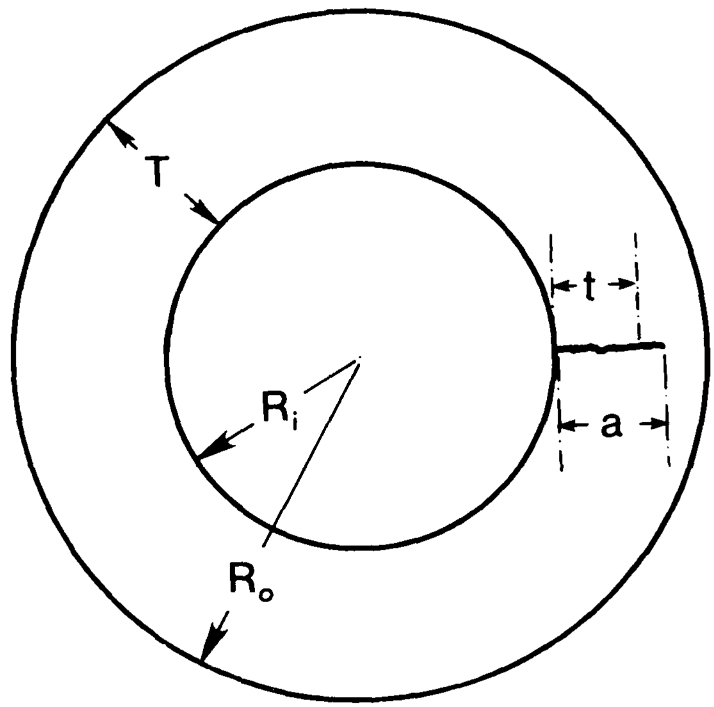

FIG. 3 - Section of a thick-walled cylinder containing a radial crack.
Ri/Ro = 0.555.

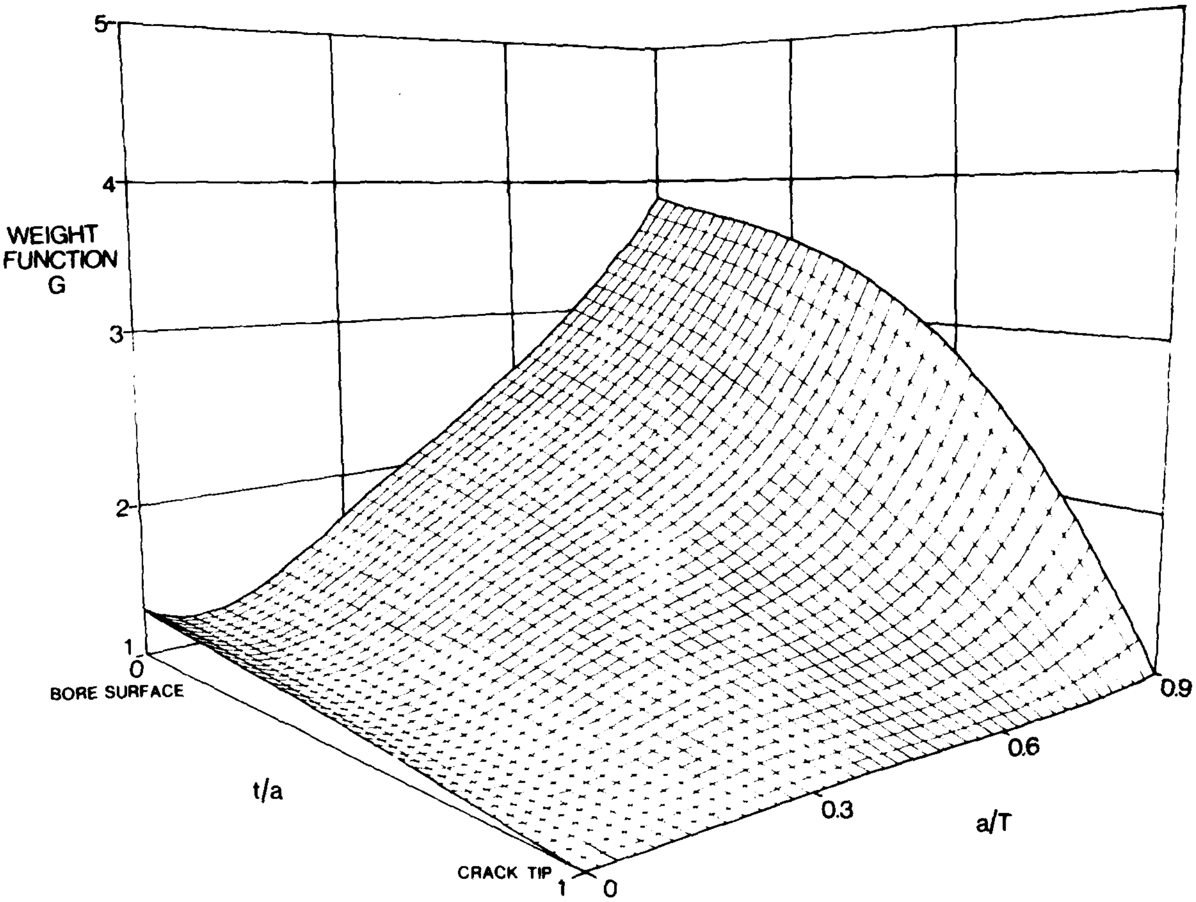

FIG. 4 - The weight function surface G(t/a, a/T) for the geometry shown in Fig. 3. The weight function is used in equation (6) to determine the stress intensity for a loading arrangement which produces a crack plane stress distribution σ(t/a) in the uncracked body.

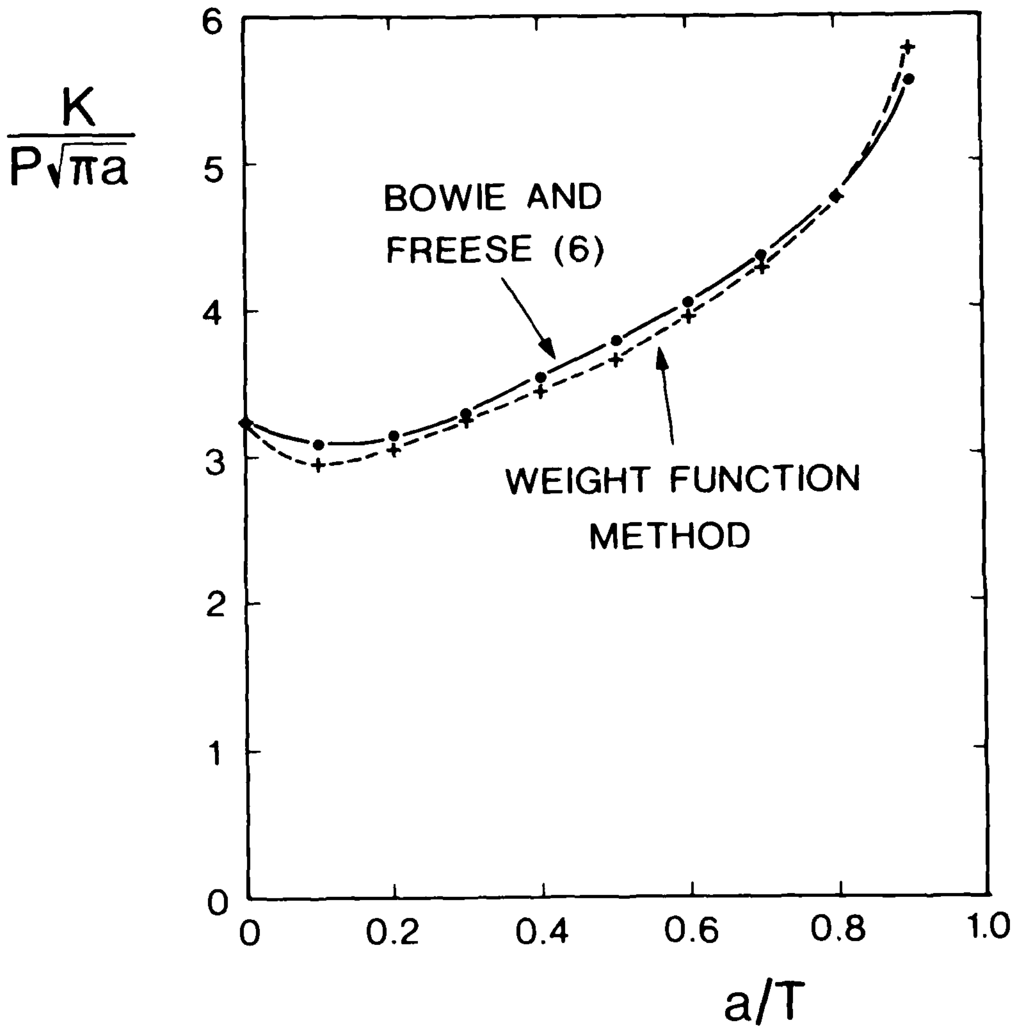

FIG. 5 - Stress intensity calibrations for an internally pressurized cylinder, with crack face pressure, derived using the weight function, and from the work of Bowie and Freese [6].

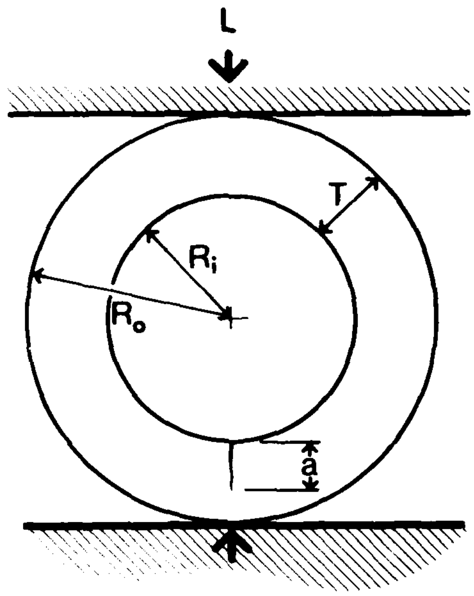

FIG. 6 - Ring compression specimen, with Ri/Ro = 0.555 and applied load L.

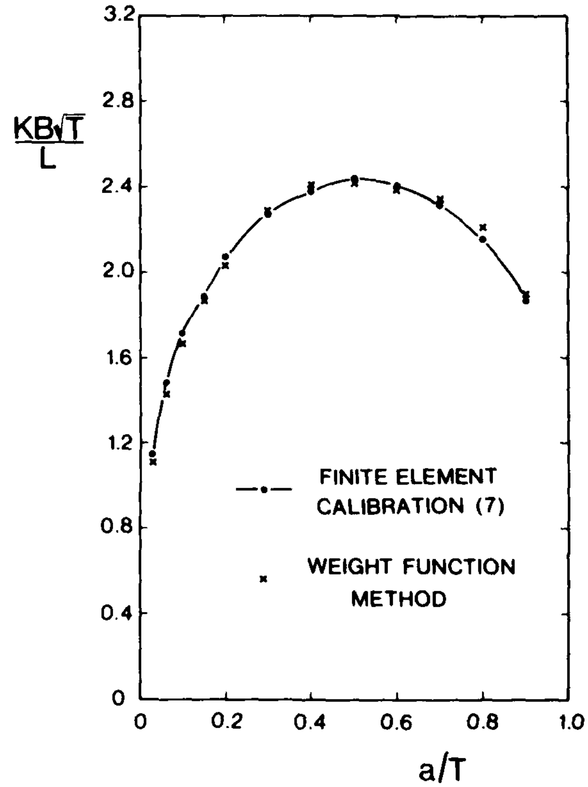

FIG. 7 - Stress intensity calibrations determined by finite element analysis [8] and using the weight function technique. Note that the weight function calibration used a surface fitted to the weight function values, rather than a more accurate representation using a separate polynomial for each crack length. The calibration is non-dimensional; B is the thickness of the ring along the cylinder axis.

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